Spinning Precessing Templates (Frequency Domain)

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`http://arXiv.org/abs/gr-qc/0508064`

Time scales characterizing spinning dynamics

- **Inspiral time scale:**

$$E = -\frac{\mu M}{2r} \qquad -\frac{dE}{dt} = \frac{32}{5}\frac{\nu^2 M^5}{r^5} \qquad \Rightarrow \qquad \frac{T_{\rm insp}}{M} \propto \frac{1}{\nu}\left(\frac{r}{M}\right)^4$$

- **Orbital time scale:**

$$\omega^2 = \frac{M}{r^3} \qquad \Rightarrow \qquad \frac{T_{\rm orb}}{M} = \frac{2\pi}{M\omega} \propto \left(\frac{r}{M}\right)^{3/2}$$

- **Precession time scale (spin-orbit coupling):** $\dot{\boldsymbol{S}} \propto \overbrace{\frac{1}{r^3}}^{\Omega^L_{\rm prec}} \boldsymbol{L} \times \boldsymbol{S}\,, \qquad \dot{\widehat{\boldsymbol{L}}} \propto \overbrace{\frac{1}{r^3}}^{\Omega^S_{\rm prec}} \boldsymbol{S} \times \widehat{\boldsymbol{L}}$

$$\frac{T^L_{\rm prec}}{M} = \frac{2\pi}{M\Omega^L_{\rm prec}} \propto \frac{1}{\nu}\left(\frac{r}{M}\right)^{5/2}\,, \qquad \frac{T^S_{\rm prec}}{M} = \frac{2\pi}{M\Omega^S_{\rm prec}} \propto \left(\frac{r}{M}\right)^3$$

$$T_{\rm orb} \ll T^L_{\rm prec}, T^S_{\rm prec} \ll T_{\rm insp}$$

Inspiraling dynamics averaging over orbital period: adiabatic limit

$$\dot{\omega} = -F(\omega)/[dE(\omega)/d\omega]$$

$$\frac{\dot{\omega}}{\omega^2} = \frac{96}{5}\,\nu\,(M\omega)^{5/3}\,\Big[1 + c_{\rm 1PN}(m_1,m_2)\,(M\omega)^{2/3} + c_{\rm 1.5PN}(SO,m_1,m_2)\,(M\omega) + c_{\rm 2PN}(SS,m_1,m_2)\,(M\omega)^{4/3} + c_{\rm 2.5PN}(SO,m_1,m_2)\,(M\omega)^{5/3} + c_{\rm 3PN}(m_1,m_2)\,(M\omega)^2 + c_{\rm 3.5PN}(m_1,m_2)\,(M\omega)^{7/3}\Big]$$

$$\dot{\boldsymbol{S}}_1 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_2}{m_1}\right)\boldsymbol{L} + \frac{1}{M^2}\left[\boldsymbol{S}_2 - 3(\boldsymbol{S}_2\cdot\hat{\boldsymbol{L}})\hat{\boldsymbol{L}}\right] + \cdots\right\}\times\boldsymbol{S}_1$$

$$\dot{\boldsymbol{S}}_2 = \frac{(M\omega)^2}{2M}\left\{\left(4+3\frac{m_1}{m_2}\right)\boldsymbol{L} + \frac{1}{M^2}\left[\boldsymbol{S}_1 - 3(\boldsymbol{S}_1\cdot\hat{\boldsymbol{L}})\hat{\boldsymbol{L}}\right] + \cdots\right\}\times\boldsymbol{S}_2\,,$$

$$\dot{\hat{\boldsymbol{L}}} = -\frac{(M\,\omega^{1/3})}{\nu\,M^2}\,\dot{\boldsymbol{S}} \qquad \boldsymbol{S} = \boldsymbol{S}_1 + \boldsymbol{S}_2$$

Two classes of binaries: single spin

- **Equal-mass binary and SS couplings neglected:**

$$\Rightarrow \boldsymbol{S} = \boldsymbol{S}_{\rm tot} = \boldsymbol{S}_1 + \boldsymbol{S}_2 \quad \text{and} \quad d(\boldsymbol{S}_1 \cdot \boldsymbol{S}_2)/dt = 0$$

- **Small mass-ratio binary:**

$$\Rightarrow m_2 \ll m_1 \quad \Rightarrow \quad |\boldsymbol{S}_2| = m_2^2\,\chi_2 \ll |\boldsymbol{S}_1| = m_1^2\,\chi_1 \quad \Rightarrow \quad \boldsymbol{S} = \boldsymbol{S}_1$$

$$\dot{\boldsymbol{S}} = \frac{\nu (M\omega)^{5/3}}{2M}\left(4 + \frac{3m_2}{m_1}\right) \hat{\boldsymbol{L}} \times \boldsymbol{S} \qquad \dot{\hat{\boldsymbol{L}}} = \frac{\omega^2}{2M}\left(4 + \frac{3m_2}{m_1}\right) \boldsymbol{S} \times \hat{\boldsymbol{L}}$$

$$\dot{\omega} = \dot{\omega}(\omega, \boldsymbol{S} \cdot \boldsymbol{L}, m_1, m_2)$$

$\alpha_{\rm prec}^J = \mathcal{B}\,\omega^{-1}$ if $L \sim \nu M^{5/3} \omega^{-1/3} \gg S$ [large separations, comparable masses]

$\alpha_{\rm prec}^J = \mathcal{B}\,\omega^{-2/3}$ if $S \gg L \sim \nu M^{5/3} \omega^{-1/3}$ [small mass ratio, last stages of inspiral]

$d\alpha_{\rm prec}^J/dt = \Omega_{\rm prec}^J$ [Apostolatos, Cutler, Sussman and Thorne 95]

Precession around the direction of the total angular momentum $\widehat{\boldsymbol{J}}$

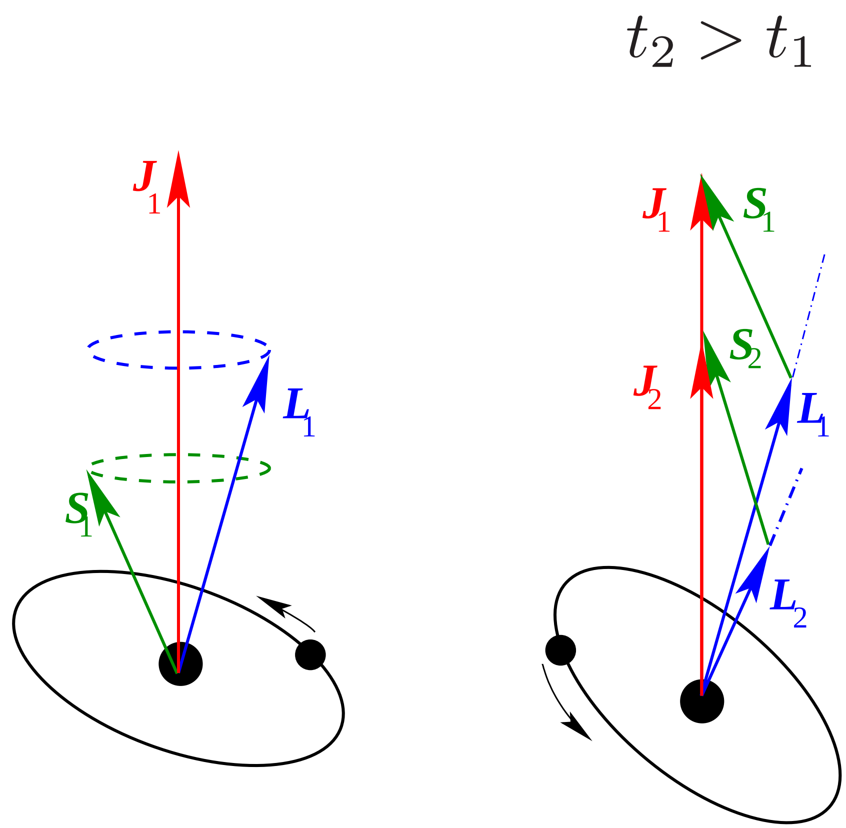
- **Single spin binary (simple precession)**

$$\dot{\boldsymbol{S}} \propto \frac{J}{r^3}\widehat{\boldsymbol{J}} \times \boldsymbol{S}$$

$$\dot{\widehat{\boldsymbol{L}}} \propto \frac{J}{r^3}\widehat{\boldsymbol{J}} \times \widehat{\boldsymbol{L}}$$

$$\Omega^J_{\rm prec} \propto \frac{J}{r^3}$$

$$\widehat{\boldsymbol{L}} \cdot \widehat{\boldsymbol{S}} = {\rm const}$$



- **Transitional precession** ($\boldsymbol{L} \simeq -\boldsymbol{S}$ the binary tumbles in space)

Gravitational waveform in the Finn-Chernoff convention

- **Assuming circular orbits, the leading order quadrupole formula reads**

$$h^{ij} = \frac{2\mu}{R} \left(\frac{M}{r}\right) Q_c^{ij} \quad \text{with} \quad Q_c^{ij} = 2\left(\lambda^i \lambda^j - n^i n^j\right)$$

$\widehat{\boldsymbol{n}} \rightarrow$ **unit vector along binary separation vector** r

$\widehat{\boldsymbol{\lambda}} \rightarrow$ **unit vector along binary relative velocity** v

$$\widehat{\boldsymbol{n}}(t) = \boldsymbol{e}_1(t)\,\cos\Phi(t) + \boldsymbol{e}_2(t)\,\sin\Phi(t) \qquad \widehat{\boldsymbol{\lambda}}(t) = -\boldsymbol{e}_1(t)\,\sin\Phi(t) + \boldsymbol{e}_2(t)\,\cos\Phi(t)$$

- **Finn-Chernoff convention:**

  – $\boldsymbol{e}_1(t) \equiv \frac{\boldsymbol{e}_z^{\mathrm{S}} \times \widehat{\boldsymbol{L}}_{\mathrm{N}}}{\sin\iota}$ and $\boldsymbol{e}_2(t) \equiv \frac{\boldsymbol{e}_z^{\mathrm{S}} - \widehat{\boldsymbol{L}}_{\mathrm{N}}\,\cos\iota}{\sin\iota}$

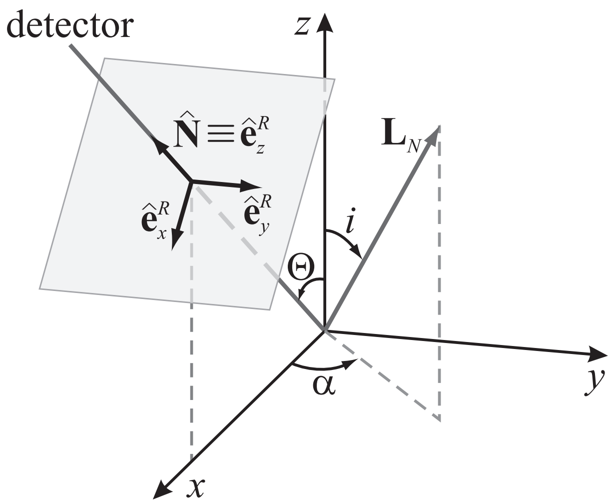
  – **the vector $\boldsymbol{e}_1$ points in the direction of the ascending node**

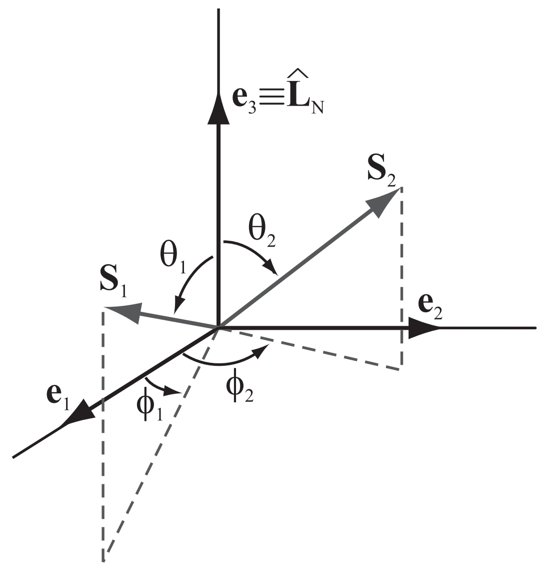
  – Φ **is the orbital phase with respect to the ascending node**

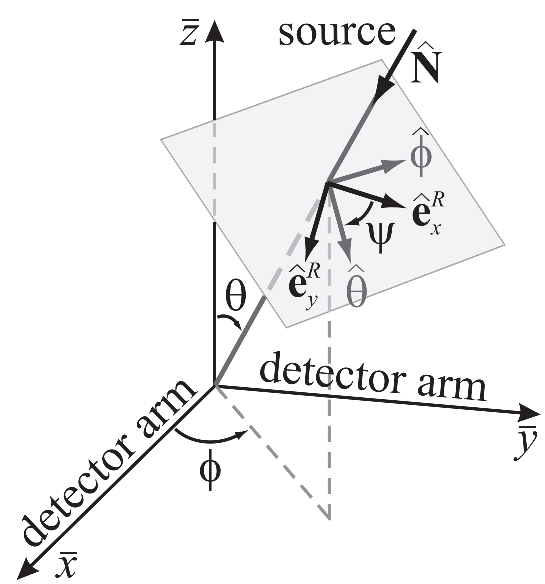
  – $\dot{\Phi} = \omega - \dot{\alpha}\,\cos\iota$ **with** $\dot{\widehat{\boldsymbol{n}}} = \omega\,\widehat{\boldsymbol{\lambda}}$

Spinning binary parameters

Binary			GW propagation	Detector orientation
M, ν, S_1, S_2	θ_{S_1}, θ_{S_2},$\phi_{S_1} - \phi_{S_2}$	$\theta_{\mathrm{L_N}} \equiv \iota$, $\phi_{\mathrm{L_N}} \equiv \alpha$, $\phi_{S_1} + \phi_{S_2}$	Θ, φ	θ,ϕ, ψ
Basic	Local	Directional		







Gravitational waveform in the Finn-Chernoff convention (continued)

$$Q_c^{ij} = -2\left(\left[\boldsymbol{e}_+^{ij}(t)\,\cos 2(\Phi(t)+\Phi_0) + \boldsymbol{e}_\times^{ij}(t)\,\sin 2(\Phi(t)+\Phi_0)\right]\right)$$

where $\boldsymbol{e}_+ \equiv \boldsymbol{e}_1 \otimes \boldsymbol{e}_1 - \boldsymbol{e}_2 \otimes \boldsymbol{e}_2$ **and** $\boldsymbol{e}_\times \equiv \boldsymbol{e}_1 \otimes \boldsymbol{e}_2 + \boldsymbol{e}_2 \otimes \boldsymbol{e}_1$

$\mathbf{T}_+ \equiv \boldsymbol{e}_x^R \otimes \boldsymbol{e}_x^R - \boldsymbol{e}_y^R \otimes \boldsymbol{e}_y^R$ **and** $\mathbf{T}_\times \equiv \boldsymbol{e}_x^R \otimes \boldsymbol{e}_y^R + \boldsymbol{e}_y^R \otimes \boldsymbol{e}_x^R$

$$\mathbf{e}_x^R = -\boldsymbol{e}_x^S\,\sin\varphi + \boldsymbol{e}_y^S\,\cos\varphi$$
$$\mathbf{e}_y^R = -\boldsymbol{e}_x^S\,\cos\Theta\cos\varphi - \boldsymbol{e}_y^S\,\cos\Theta\,\sin\varphi + \boldsymbol{e}_z^S\,\sin\Theta$$
$$\mathbf{e}_z^R = +\boldsymbol{e}_x^S\,\sin\Theta\,\cos\varphi + \boldsymbol{e}_y^S\,\sin\Theta\,\sin\varphi + \boldsymbol{e}_z^S\,\cos\Theta = \mathbf{N}$$

$$h = h_+\,F_+ + h_\times\,F_\times = \frac{2\mu}{R}\,\frac{M}{r}\,Q_c^{ij}\,([\mathbf{T}_+]_{ij}\,F_+ + [\mathbf{T}_+]_{ij}\,F_+)$$

Phenomenological templates in adiabatic limit: Apostolatos ansatz

Phenomenological waveforms: introducing few *new* parameters and having reasonably good matches with the signal

Apostolatos' ansatz: add modulations to SPA phase [Apostolatos 96]

$$\psi_{\rm SPA}(f) = f^{-5/3}\,\left(\psi_0 + \psi_1\, f^{2/3} + \psi_{3/2}\, f + \cdots\right) + \mathcal{C}\,\cos(\delta + \alpha^J_{\rm prec})$$

$$\alpha^J_{\rm prec} = \mathcal{B} f^{-2/3} \quad \text{or} \quad \mathcal{B} f^{-1} \qquad \text{where} \qquad d\alpha^J_{\rm prec}/dt = \Omega^J_{\rm prec}$$

– **Not very satisfactory performances (why?)** [Grandéclement, Kalogera & Vecchio 96]

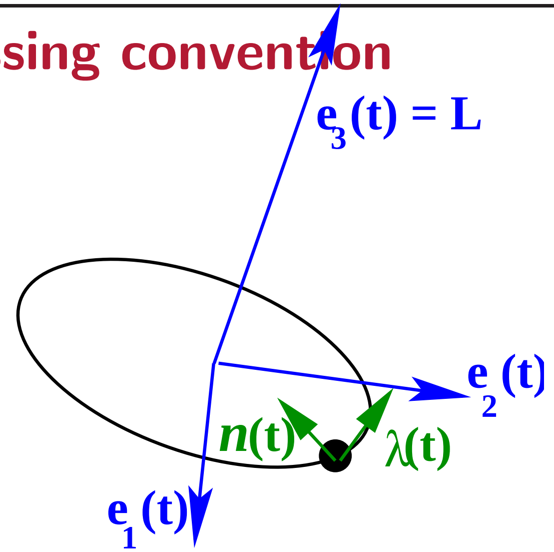
– **three** *new* **intrinsic parameters** $(m_1, m_2, \mathcal{C}, \delta, \mathcal{B})$

– **high computational cost**

New convention for the generation and propagation of GWs from spinning binaries

This convention has the desirable property of factorizing the waveform into a *carrier signal* **whose phase is essentially the accumulated orbital phase of the binary, and a** *modulated signal* **term which is sensitive to the precession of the orbital plane**

Gravitational waveform in precessing convention



[AB, Chen & Vallisneri 03]

$$h^{ij} \propto \ddot{Q}^{ij}\,, \qquad \ddot{Q}^{ij} = 2(\lambda^i\lambda^j - n^i n^j)$$

$$\hat{\boldsymbol{n}}(t) = \boldsymbol{e}_1(t)\,\cos\Phi(t) + \boldsymbol{e}_2(t)\,\sin\Phi(t)$$
$$\hat{\boldsymbol{\lambda}}(t) = -\boldsymbol{e}_1(t)\,\sin\Phi(t) + \boldsymbol{e}_2(t)\,\cos\Phi(t)$$

- **adiabatic sequence of spherical orbits** $\Rightarrow \dot{\hat{\boldsymbol{n}}} = \omega\hat{\boldsymbol{\lambda}}$ **but in general** $\dot{\Phi} \neq \omega$

- ω **almost non-modulated**

Precessing convention: $\dot{e}_i(t) = \Omega_e(t) \times e_i(t)$ **such that** $\dot{\Phi} = \omega$ **!**

$$\boldsymbol{\Omega}_e(t) \equiv \boldsymbol{\Omega}_L(t) - [\boldsymbol{\Omega}_L(t)\cdot\hat{\boldsymbol{L}}_N(t)]\,\hat{\boldsymbol{L}}_N(t)\,, \quad \dot{\hat{\boldsymbol{L}}}_N = \boldsymbol{\Omega}_L \times \hat{\boldsymbol{L}}_N(t)\;, \;\; \boldsymbol{\Omega}_e\cdot\hat{\boldsymbol{L}}_N = 0$$

Gravitational waveform in precessing convention (continued)

[Apostolatos 95; AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri 05]

Clean separation of time-dependent and time-independent parameters in waveforms

$$h(t) = \underbrace{-\frac{2\mu}{R}\frac{M}{r(t)}\left[\boldsymbol{e}_{+}^{ij}(t)\,\cos 2(\Phi(t)+\Phi_0) + \boldsymbol{e}_{\times}^{ij}(t)\,\sin 2(\Phi(t)+\Phi_0)\right]}_{\mathrm{Q(t):\,wave\,generation}} \times$$

$$\underbrace{[T_{+\,ij}(\Theta,\varphi)\,F_{+}(\theta,\phi,\psi) + T_{\times\,ij}(\Theta,\varphi)\,F_{\times}(\theta,\phi,\psi)]}_{\mathrm{P:\,detector\,projection}}$$

$P_{ij} \;\Rightarrow\;$ *static* **relative position and orientation of detector and wave with repect to the axes initially defined by the binary**

$$\tilde{h}_{\mathrm{SPA}}(f) = -\tilde{h}^{\mathrm{carrier}}_{\mathrm{non\,mod}}(f)\left([\mathbf{e}_{+}(t_f)]^{jk} + i\,[\mathbf{e}_{\times}(t_f)]^{jk}\right)\,([\mathbf{T}_{+}]_{jk}\,F_{+} + [\mathbf{T}_{\times}]_{jk}\,F_{\times})$$

Comparison between different conventions

Binary			GW propagation	Detector orientation
M, ν, S_1, S_2	θ_{S_1}, θ_{S_2},$\phi_{S_1} - \phi_{S_2}$	$\theta_{\mathrm{L_N}} \equiv \iota$, $\phi_{\mathrm{L_N}} \equiv \alpha$, $\phi_{S_1} + \phi_{S_2}$	Θ, φ	θ,ϕ, ψ
Basic	Local	Directional		

convention	factor P ($\mathbf{T}_{+,\times}$ $F_{+,\times}$)	factor Q $\Phi(t)$	factor Q $\mathbf{e}_{+,\times}(t)$
ACST	function of basic, local, and directional parameters; time dependent	function of basic, local, and directional parameters	function of basic, local, and directional parameters
FC	function of directional parameters; time independent	function of basic, local, and directional parameters	function of basic, local, and directional parameters
precessing	function of directional parameters; time independent	function of basic and local parameters only; coincides with $\Psi(t) = \int \omega \, dt$	function of basic and local parameters only

Templates for spinning, precessing binaries

[Apostolatos 95; AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri 05]

Clean separation of time-dependent and time-independent parameters in waveforms

$$h(t) \propto \underbrace{\frac{1}{r(t)} \left[\boldsymbol{e}_{+}^{ij}(t)\, \cos 2(\Phi(t) + \Phi_0) + \boldsymbol{e}_{\times}^{ij}(t)\, \sin 2(\Phi(t) + \Phi_0) \right]}_{\mathrm{Q(t): wave\ generation}} \times$$

$$\underbrace{[P_{ij}(\Theta, \varphi, \theta, \phi, \psi)]}_{\mathrm{P: detector\ projection}}$$

$$\tilde{h}_{\mathrm{SPA}}(f) = \mathcal{A}\, f^{-7/6}\, e^{i\psi_{\mathrm{SPA}}(f) + i\Phi_0 + 2\pi i f t_0} \times$$

$$\left[1 + \mathcal{C}_{\cos}\, e^{i\phi_{\cos}}\, \cos(\alpha_p(f)) + \mathcal{C}_{\sin}\, e^{i\phi_{\sin}}\, \sin(\alpha_p(f))\right]$$

$$\psi_{\mathrm{SPA}}(f) \equiv \psi_{\mathrm{SPA}}(f, m_1, m_2, \mathbf{S}\cdot\mathbf{L}, \chi) \qquad \alpha_p(f) \equiv \alpha_p(f, m_1, m_2, \mathbf{S}\cdot\mathbf{L}, \chi)$$

Very fast (analytic) maximization on $\{t_0, \Phi_0, \phi_{\cos}, \phi_{\sin}, \mathcal{C}_{\cos}, \mathcal{C}_{\sin}\}$ **!**

Phenomenological frequency-domain template family

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri 05]

$$h(f) = \mathcal{A}(f)\, e^{i\psi(f)}$$

$$\mathcal{A}(f) = f^{-7/6}\,\theta(f_{\rm cut} - f)\times$$

$$[(\mathcal{C}_1 + i\mathcal{C}_2) + (\mathcal{C}_3 + i\mathcal{C}_4)\,\cos\,(\mathcal{B} f^{-p}) + (\mathcal{C}_5 + i\mathcal{C}_6)\,\sin\,(\mathcal{B} f^{-p})]$$

$$\psi(f) = f^{-5/3}\,\left(\psi_0 + \psi_{1/2}\, f^{1/3} + \psi_1\, f^{2/3} + \psi_{3/2}\, f + \cdots\right) + 2\pi f\, t_0$$

To get high signal matches, sufficient to set $\psi_0, \psi_{3/2} \neq 0$ and all $\psi_i = 0$

- 3 intrinsic parameters $(\psi_0, \psi_{3/2}, \mathcal{B}) + f_{\rm cut}$ and 7 extrinsic parameters $(\mathcal{C}_1, \mathcal{C}_2, \ldots)$
- Fast search on the seven extrinsic parameters $\Rightarrow$ low computational cost

Intrinsic and extrinsic parameters

- *Intrinsic parameters*: **variables on which we lay down templates**

- *Extrinsic parameters*: **variables that can be eliminated from the template space**
  $\Rightarrow$ **we don't need to lay down templates along those directions**

- **Time of arrival t_a: all possible shifts in time can be obtained at once with a single anti-Fourier transform**

$$(o, h(\lambda; t_a)) = 4\mathrm{Re}\left[\int_0^{+\infty} df\, \frac{\tilde{h}^*(f;\lambda)\, \tilde{o}(f)\, e^{2\pi i\, f\, t_a}}{S_n(f)}\right]$$

  We locate the value of t_a for which $(o, h(\lambda; t_a))$ is maximum

- **The overall amplitude is not relevant when searching for the template that maximizes the statistic**

Extrinsic parameters (continued)

- **Maximization over the phase Φ_0 can be done analytically**

$$h(\lambda; \Phi_0) = \mathcal{A}_\lambda(t)\, \cos\left[\Phi_\lambda(t) + \Phi_0\right]$$

For each $\{\mathcal{A}_\lambda(t), \Phi_\lambda(t)\}$ we need to keep in the template bank only two copies of h

i.e., h_0 and $h_{\pi/2}$ $\Rightarrow (o, h_{\Phi_0}) = \cos\Phi_0\,(o, h_0) + \sin\Phi_0\,(o, h_{\pi/2})$

- **What is $\max_{\Phi_0}(o, h(\Phi_0))$ with $(h, h) = 1$?**

We first orthonormalize h_0 and $h_{\pi/2}$ $\Rightarrow (\hat{h}_0, \hat{h}_0) = (\hat{h}_{\pi/2}, \hat{h}_{\pi/2}) = 1\,,\ (\hat{h}_0, \hat{h}_{\pi/2}) = 0$

$$\Rightarrow \rho_{\Phi_0} \equiv \max_{\Phi_0}(o, h(\Phi_0)) = \sqrt{(o, \hat{h}_0)^2 + (o, \hat{h}_{\pi/2})^2}$$

- **The phase-maximized statistic for the case of Gaussian noise ($o = n$) is the Raleigh distribution** $\mathcal{P}(\rho_{\Phi_0}) = \rho_{\Phi_0}\, e^{-\rho^2_{\Phi_0}/2}$

Extrinsic parameters (continued)

- **More in general if we have α_k parameters such that**

$$(o, h(\lambda; \alpha_k)) = \sum_k \hat{\alpha}_k \,(o, \hat{h}_k) \quad \text{with} \quad \sum_k \hat{\alpha}_k^2 = 1$$

by interpreting it as a scalar product in Euclidean space, the maximum is obtained when the vector $\{\hat{\alpha}_k\}$ is parallel to $(o, \hat{h}_k)/\sqrt{\sum_k (o, \hat{h}_k)^2}$

$$\Rightarrow \quad \hat{\alpha}_k = (o, \hat{h}_k)/\sqrt{\sum_k (o, \hat{h}_k)^2}$$

$$\Rightarrow \quad \rho_{\alpha_k} \equiv \max_{\alpha_k} (o, h(\alpha_k)) = \sqrt{\sum_k (o, \hat{h}_k)^2}$$

- **The maximized statistic for the case of Gaussian noise ($o = n$) is the χ distribution with $2n = k$ degrees of freedom** $\mathcal{P}(\rho_{\alpha_k}) = \rho_{\alpha_k}^{2n-1}\, e^{-\rho_{\alpha_k}^2/2}/(2^{n-1}\,\Gamma(n))$

When do we really gain?

$\mathcal{N}_{\rm shapes}$	Threshold for false-alarm probability $= 10^{-3}$		
	$(\psi_0\, \psi_{3/2})_2$ $n = 1$	$(\psi_0\, \psi_{3/2}\, \alpha)_4$ $n = 2$	$(\psi_0\, \psi_{3/2}\, \mathcal{B})_6$ $n = 3$
10^2	8.44	8.87	9.22
10^3	8.71	9.13	9.48
10^4	8.97	9.39	9.73
10^5	9.22	9.63	9.97
10^6	9.47	9.87	10.21

Table 1: Detection thresholds for a false-alarm probability $= 10^{-3}$ for a χ-distributed detection statistic with $2n$ degrees of freedom, for $\mathcal{N}_{\rm times} = 3\, 10^{10}$, and for the $\mathcal{N}_{\rm shapes}$ given in the first column.

Performances for high mass and small mass-ratio binaries

Averaging over uniform distribution of $S_{1,2}$ and L directions

	High-mass binaries				
	$(7+5)M_\odot$ $\overline{\mathrm{FF}}$	$(10+10)M_\odot$ $\overline{\mathrm{FF}}$	$(15+15)M_\odot$ $\overline{\mathrm{FF}}$	$(20+5)M_\odot$ $\overline{\mathrm{FF}}$	$(20+10)M_\odot$ $\overline{\mathrm{FF}}$
non-mod SPA	0.903	0.894	0.811	0.858	0.826
$(\psi_0\psi_{3/2}\mathcal{B}f_{\mathrm{cut}})$	0.975	0.986	0.986	0.974	0.984

Increase of overlap can be lower than increase in threshold but templates are closer to signal

	Small mass-ratio and equal–low-mass binaries					
	$(10+1.4)M_\odot$ $\overline{\mathrm{FF}}$	$(12+1)M_\odot$ $\overline{\mathrm{FF}}$	$(12+2)M_\odot$ $\overline{\mathrm{FF}}$	$(6+3)M_\odot$ $\overline{\mathrm{FF}}$	$(6+1)M_\odot$ $\overline{\mathrm{FF}}$	$(2+2)M_\odot$ $\overline{\mathrm{FF}}$
non-mod SPA	0.780	-	-	-	-	-
$(\psi_0\psi_{3/2}\mathcal{B})$	0.933	0.932	0.960	0.975	0.937	0.964

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri 05]

Physical frequency-domain template family (single spin)

[AB, Chen, Pan & Vallisneri 05]

- The three *phenomenological* parameters $(\psi_0, \psi_{3/2}, \mathcal{B})$ traded with four *physical* parameters (M, ν, κ, χ)

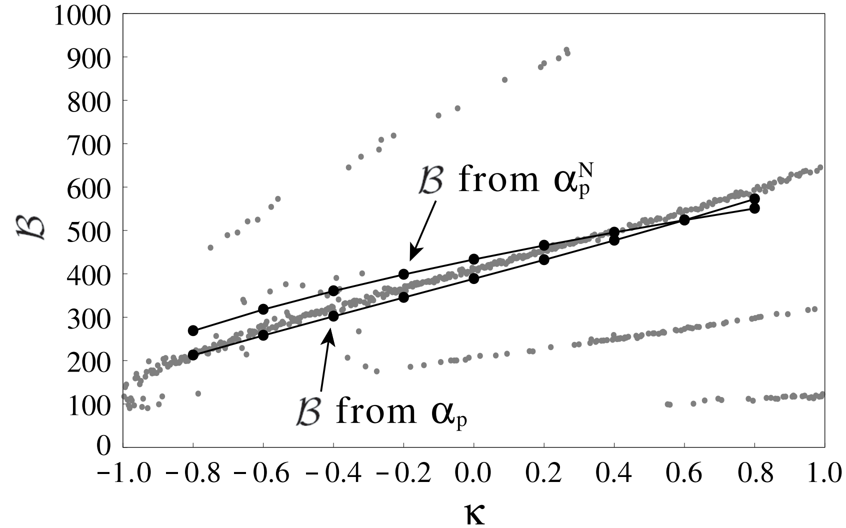
- $\tilde{Q}_c^{ij}(f) \sim \left[\mathcal{C}_0^{ij} + \cos(\delta^{ij} + \alpha^J_{\rm prec}(M,\nu,\kappa,\chi))\,\mathcal{C}_1^{ij}\right] \; f^{-7/6}\, e^{i\psi_{\rm SPA}(M,\nu,\kappa,\chi)}$

- $\alpha^J_{\rm prec}(M,\nu,\kappa,\chi)$ **known analytically !**

  $\alpha^J_{\rm prec} \sim \mathcal{B}\, f^{-p}$

— **The template metric *is not* analytical**

— **4D template space**



Systematic errors: $\Delta\mathcal{M}/\mathcal{M} \sim 1\%$, $\Delta M/M, \Delta\nu/\nu \sim 10\%$, $\Delta\chi/\chi \sim 20\%$, $\Delta\kappa \sim 2\%$

Physical frequency-domain template family (single spin) (continued)

$$\Omega_{\rm prec} = \left(2 + \frac{3m_2}{2m_1}\right)\, \sqrt{1 + 2\kappa\gamma + \gamma^2}\,\frac{L}{r^3}\,, \quad \gamma = \frac{S}{L}\,, \quad \kappa = \hat{\boldsymbol{L}}\cdot\hat{\boldsymbol{S}}$$

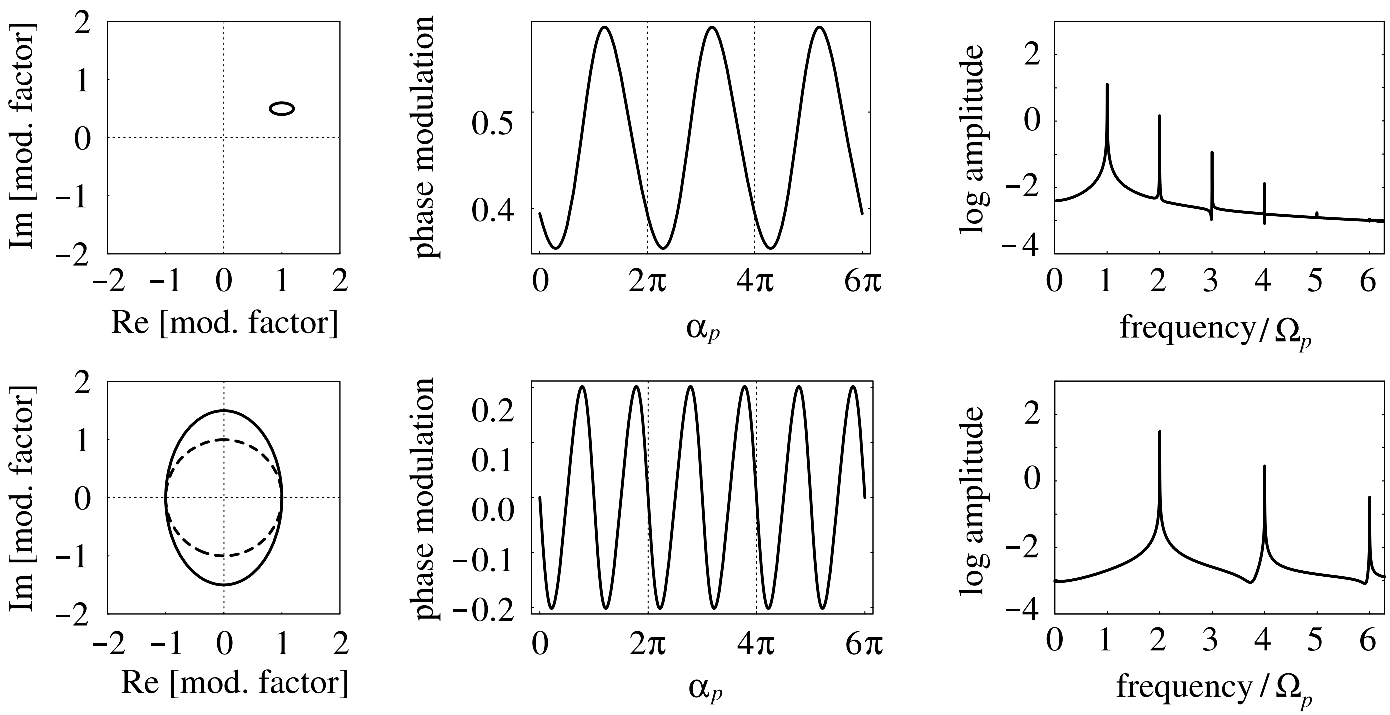
integrating

$$\alpha^{\rm Newt}_{\rm prec}(f) = \frac{5}{384}\frac{4m_1+3m_2}{m_1} \times \left\{ -A\, \left[(2-3\kappa^2)\,\chi_M^2 + \kappa\,\chi_M\, v^{-1} + 2\,v^{-2}\right] + 3\,\kappa\,(1-\kappa^2)\,\chi_M^3\, \log\left[\pi^{1/3}\eta\left(\kappa\,\chi_M + v^{-1} + A\right)\right]\right\}$$

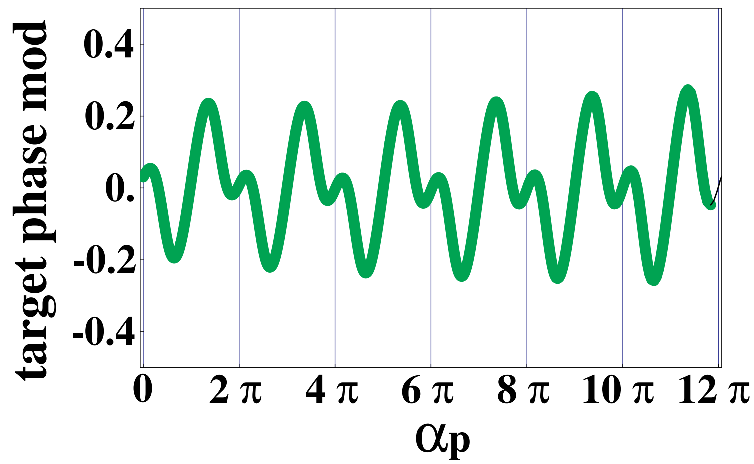
$$v = (\pi M f)^{1/3}\,, \quad \chi_M = \frac{m_1}{m_2}\chi\,, \quad A = \sqrt{\chi_M^2 + 2\,\kappa\,\chi_M\, v^{-1} + v^{-2}}$$

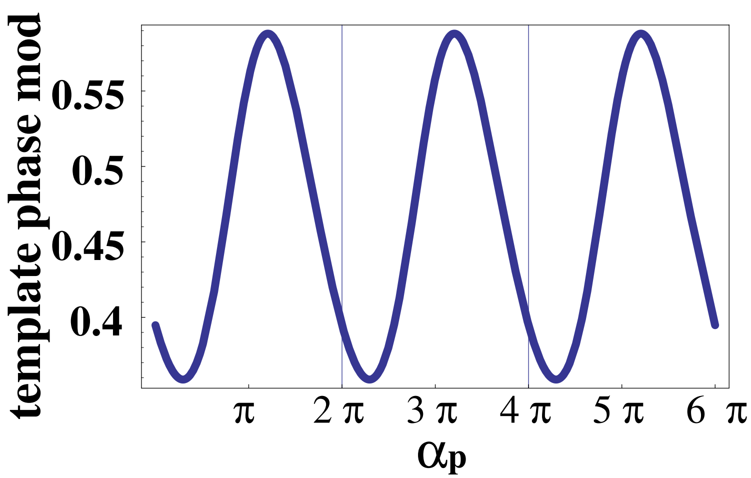
Higher harmonics are present!

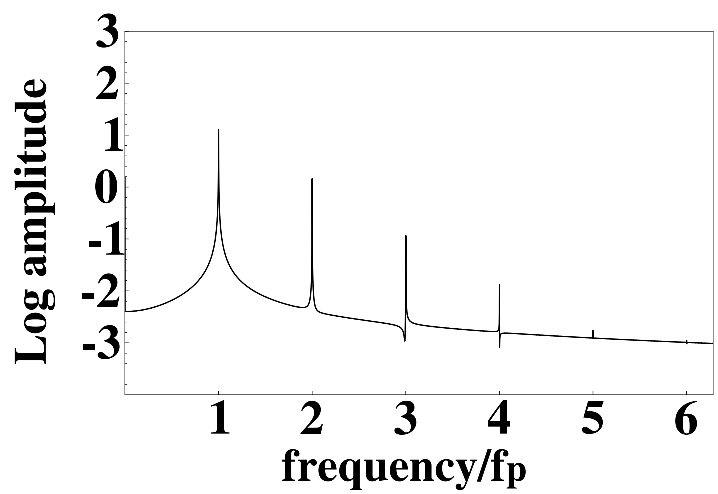
$$\left[(\mathcal{C}_1 + i\mathcal{C}_2) + (\mathcal{C}_3 + i\mathcal{C}_4)\,\cos\left(\mathcal{B} f^{-p}\right) + (\mathcal{C}_5 + i\mathcal{C}_6)\,\sin\left(\mathcal{B} f^{-p}\right)\right]$$



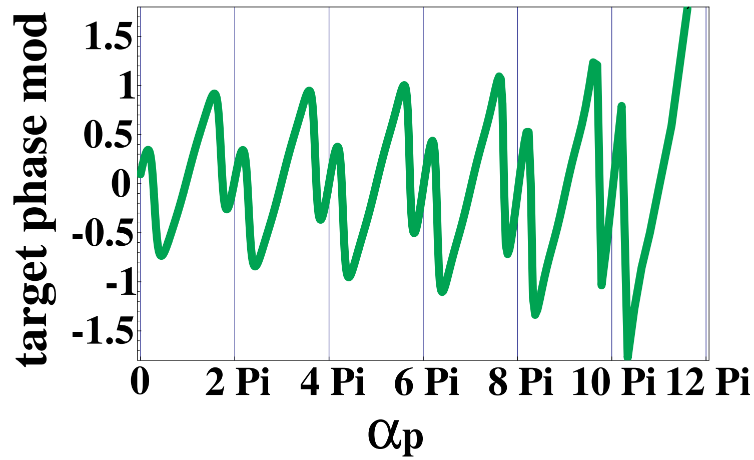
Phase modulations in target and template: harmonics

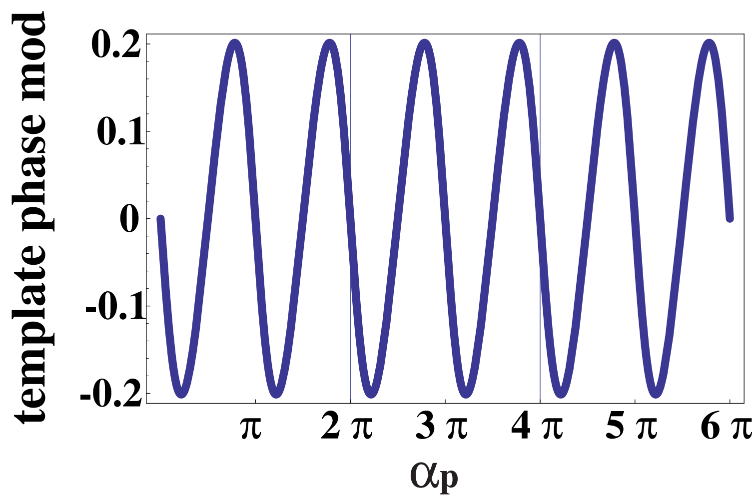


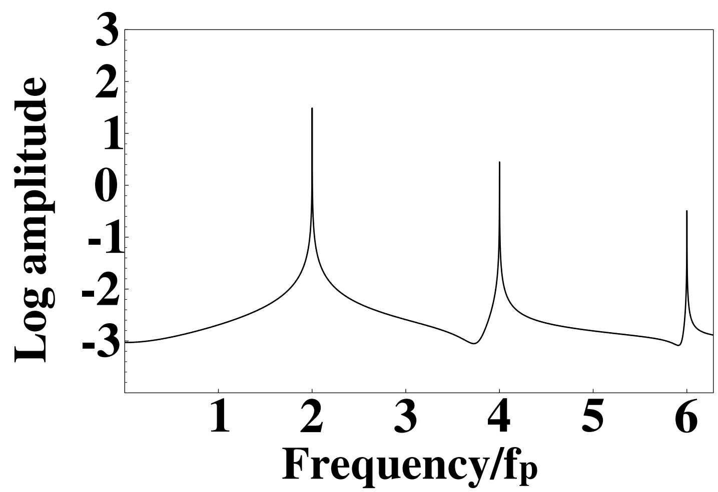




Phase modulations in target and template: harmonics (continued)

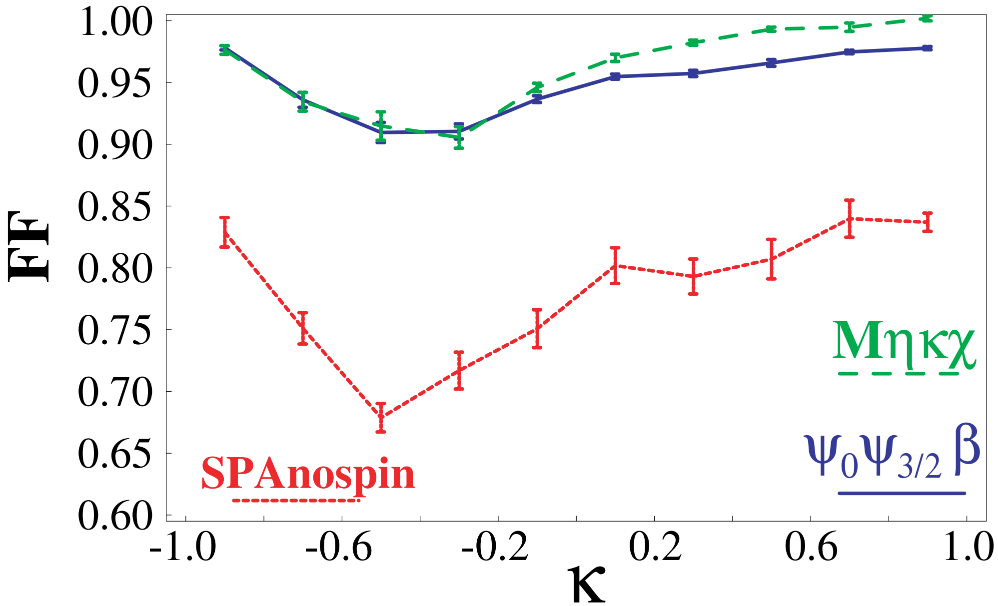






Performances for NS/BH binary $(10 + 1.4)M_\odot$

[AB, Chen & Vallisneri 03; AB, Chen, Pan & Vallisneri 05]



Mismatch template metric and projected metric

[Balasubramanian, Dhurandhar, Sathyaprakash 91, 94, 96; Owen 96]

- $1 - \langle \hat{h}(\lambda^A), \hat{h}(\lambda^A + \Delta\lambda^A) \rangle \equiv \delta[\lambda^A, \lambda^A + \Delta\lambda^A] = g_{BC} \Delta\lambda^B \Delta\lambda^C$

$$g_{BC} = -\frac{1}{2} \frac{\partial^2 \left\langle \hat{h}(\lambda^A), \hat{h}(\lambda^A + \Delta\lambda^A) \right\rangle}{\partial(\Delta\lambda^B)\partial(\Delta\lambda^C)}$$

[Pan, AB, Chen & Vallisneri 04; AB, Chen, Pan, Tagoshi & Vallisneri 05]

$X^\alpha \Rightarrow$ **intrinsic parameters** $(\psi_0, \psi_3, \mathcal{B})$, $\Xi^\alpha \Rightarrow$ **extrinsic parameters** $(\mathcal{C}_i, t_0)$

- $\delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) = \begin{pmatrix} \Delta X^i & \Delta\Xi^\alpha \end{pmatrix} \begin{pmatrix} G_{ij} & C_{i\beta} \\ C_{\alpha j} & \gamma_{\alpha\beta} \end{pmatrix} \begin{pmatrix} \Delta X^j \\ \Delta\Xi^\beta \end{pmatrix}$

- $\min_{\Delta\Xi^\alpha} \delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) = \left[G_{ij} - C_{i\alpha}(\gamma^{-1})^{\alpha\beta} C_{\beta j} \right] \Delta X^i \Delta X^j$

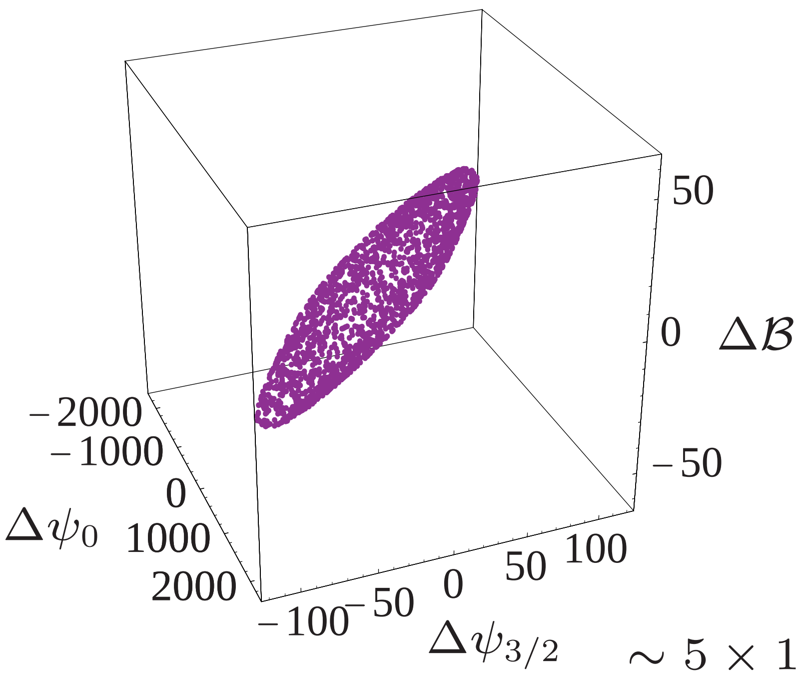
$\equiv g^{\mathrm{proj}}_{ij} \Delta X^i \Delta X^j$ $\quad$ g^{proj}_{ij} **depends only on** $\mathcal{B}$ **and** $\mathcal{C}_i$

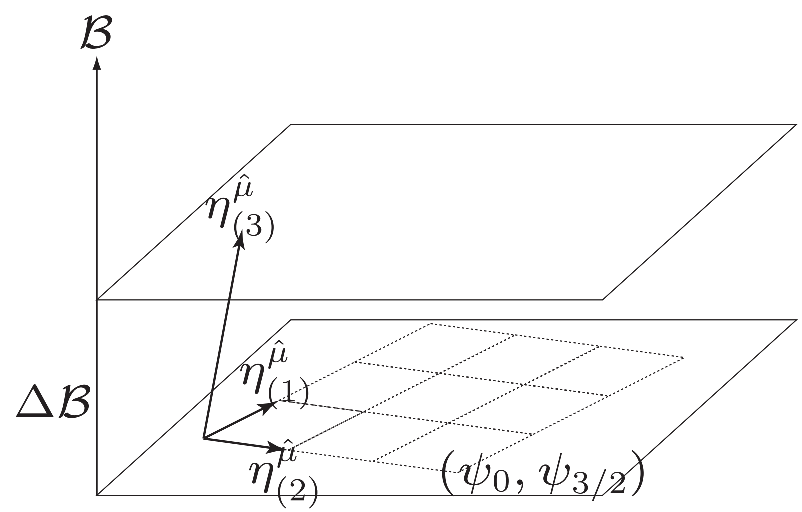
Projected metric (continued)

[Damour, Iyer and Sathyaprakash 98]

[Pan, AB, Chen & Vallisneri 04; AB, Chen, Pan, Tagoshi & Vallisneri 05]

$$\max_{\Xi^\alpha} \min_{\Delta\Xi^\alpha} \delta(X^i, \Xi^\alpha; X^i + \Delta X^i, \Xi^\alpha + \Delta\Xi^\alpha) \simeq \hat{g}^{\rm proj}_{ij}(\mathcal{B})\Delta X^i \Delta X^j = 1 - {\rm MM}$$





$\sim 5 \times 10^5$ **templates for** $m_1 = 6$–$12 M_\odot$ **and** $m_2 = 1$–$3 M_\odot$

Proposal

Can we build a search for Advanced LIGO targeted to NS-BH and BH-BH with mass ratio $6-12$ using (frequency-domain) physical templates discussed above?