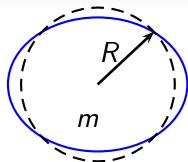


EOS EFFECTS IN THE EARLY NS INSPIRAL

- Equation of State (EoS)- dependent tidal deformation
- Small but robust signature in the waveform
- Characterized by the tidal deformability λ

$$\lambda = - \frac{\text{(induced quadrupole)}}{\text{(perturbing tidal field)}}$$

TIDAL DEFORMABILITY λ



- Equilibrium star placed in a tidal field \mathcal{E}_{ij}
- Linear response: $Q_{ij} = -\lambda \mathcal{E}_{ij}$
(induced quadrupole)

Q_{ij}, \mathcal{E}_{ij} Defined via the gravitational potential outside the star:

$$\begin{aligned}\frac{(1 + g_{tt})}{2} &\approx \frac{m_1 + m_2}{r} + \frac{1}{2} \left[-\frac{3Q_{ij}}{r^5} + \mathcal{E}_{ij} \right] x^i x^j + \dots \\ &\approx \frac{m_1 + m_2}{r} + \frac{1}{2} \left[2k_2 \frac{R^5}{r^5} + 1 \right] \mathcal{E}_{ij} x^i x^j + \dots\end{aligned}$$

$$\mathcal{E}_{ij} x^i x^j \sim \frac{m_2}{r^3} Y_{20}(\theta, \phi) r^2$$

$$\lambda = \frac{2}{3} k_2 R^5$$

dimensionless Love number

radius of the star

COMPUTATION OF λ

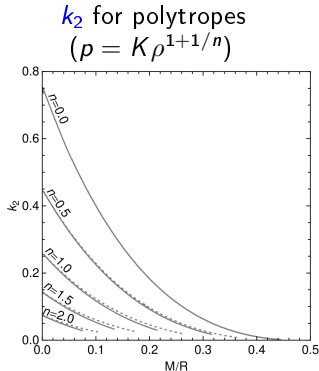
Linear perturbations lead to a master equation for $\delta g_{tt} = H(r)Y_{20}(\theta, \phi)$

- **Inside** the NS: numerically integrate
- **Outside**: perturbed Schwarzschild with asymptotics

$$H \sim (\lambda \mathcal{E})/r^3 \text{ \& \ } \mathcal{E} r^2$$

- **Match** H and H' at $r = R$,
Result:

$$\lambda = \frac{2}{3} k_2 \left(\frac{M}{R}, \frac{RH'}{H} \right) R^5$$



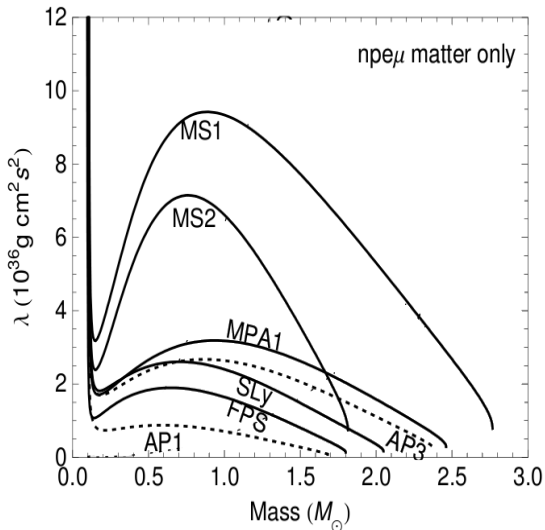
λ FOR VARIOUS EOS MODELS

Feature of NS EoS models:

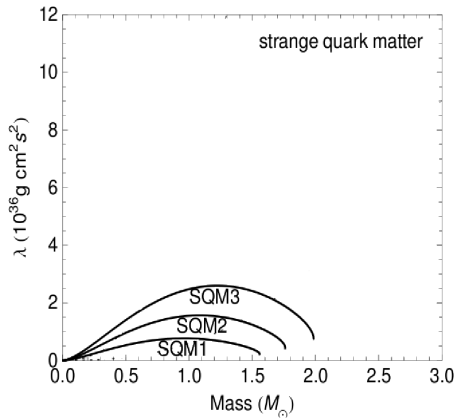
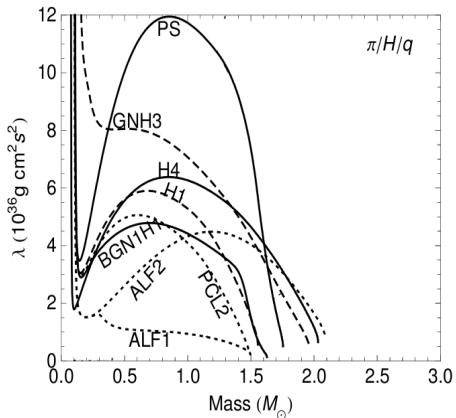
fixed relationship $R(m)$

$\lambda = \lambda(m)$ for a given EoS,

for a black hole: $\lambda = 0$



λ FOR VARIOUS EOS MODELS



MODELLING THE NS BINARY DYNAMICS

- post-Newtonian formalism (weak field, slow motion)
- internal quadrupolar degrees of freedom of each star parameterized by Q_{ij}
- coupling to the companion's $\ell = 2$ tidal field
- leading order GW dissipation terms

▷ Equations of motion for relative distance \mathbf{x} , Q_{ij} :

$$\ddot{\mathbf{x}}_i = a_i^{\text{Newt}} [1 + (\text{PN})] + a_i^{\text{tidal}} [1 + (\text{PN})] - x^j a_{ij}^{\text{RR}},$$

$$\ddot{Q}_{ij} + \omega_0^2 Q_{ij} = \lambda \omega_0^2 \mathcal{E}_{ij} - \lambda \omega_0^2 a_{ij}^{\text{RR}}$$

▷ Total quadrupole: $Q_{ij}^{\text{system}} = Q_{ij} + Q_{ij}^{\text{orbit}}$

EFFECTIVE DESCRIPTION

Timescales : ω_0^{-1} – internal f – modes
 ω^{-1} – orbital
 t_{rr} – gravitational radiation reaction

Adiabatic limit: $t_{\text{rr}} \gg \omega^{-1}$, ω_0^{-1} , solutions for quasi-circular orbits:

$$E(\omega) = -\frac{\mu}{2}(M\omega)^{2/3} \left[1 + \text{PN} - \lambda_1 \delta E(1 + \text{PN}) + 1 \leftrightarrow 2 \right],$$

$$\dot{E}(\omega) = -\frac{32}{5}(\mu M^2 \omega^5)^2 \left[1 + \text{PN} + \lambda_1 \delta \dot{E}(1 + \text{PN}) + 1 \leftrightarrow 2 \right]$$

Fourier transform of the GW signal:

$$\tilde{h}(f) = \mathcal{A}e^{i\Psi}$$

Obtain the phase from energy balance:

$$d^2\Psi/d\omega^2 = 2(dE/d\omega)/\dot{E}$$

EFFECT ON THE GW SIGNAL

$$\tilde{h}(f) = \frac{Q(\alpha, \delta, \iota, \psi)}{D_L} \mathcal{M}^{5/6} f^{-7/6} (1 + \text{PN}) e^{i\Psi}$$

Further adiabatic approx. : $\omega \ll \omega_0$, simple tidal phase contribution:

$$\begin{aligned} \Psi \sim & 2\pi f t_c - \phi_c + \frac{3}{4} (8\pi \mathcal{M} f)^{-5/3} \left\{ 1 + \frac{20}{9} \left(\frac{743}{336} + \frac{11}{4} \eta \right) x + O(x^{3/2}) \right. \\ & \left. - \lambda_1 \frac{x^5}{M^4 m_2} \left[24 \left(1 + 11 \frac{m_1}{M} \right) \right. \right. \\ & \left. \left. + \frac{5x}{28} \left(3179 - 912 \frac{m_2}{M} - 2286 \frac{m_2^2}{M^2} + 260 \frac{m_2^3}{M^3} \right) \right] + (1 \leftrightarrow 2) \right\} \end{aligned}$$

$$\text{identical stars : } \Psi^{\text{tidal}} = \frac{3}{32x^{5/2}} \left[-624 \frac{x^5}{M^5} \lambda \left(1 + \frac{3115}{1248} x \right) \right]$$

$$M = m_1 + m_2, \quad x = (M\omega)^{2/3}, \quad \eta = m_1 m_2 / M^2, \quad \mathcal{M} = (m_1 m_2)^{3/5} M^{-1/5}$$

ACCURACY OF THE PHASING MODEL

- Compute fractional corrections to the phasing model by relaxing the approximations made.
- At $f = 450\text{Hz} \ll f_{\text{isco}}$, these are $< 10\%$:

▷ Adiabatic approx : (i) $\omega^{-1} \ll t_{\text{rr}} : \lesssim 0.01$

(ii) $\omega \ll \omega_0 : \approx 0.012 (f/450\text{Hz})$

▷ Higher multipoles: $\approx 0.007 (f/450\text{Hz})^{4/3}$

▷ Nonlinear effects : (i) nonlin. response : $\approx 0.03 (f/450\text{Hz})^{5/3}$

(ii) nonlin. hydro : $\approx 0.002 (f/450\text{Hz})^2$

▷ Spin: $\lesssim 0.003$, viscosity: negligible

▷ Higher PN: 1PN was dominant : $\sim 18\%$, 1PN now included .